

Model Answer**AS-2340****M.C.A. (First Semester) EXAMINATION, 2013
Programming Based Numerical Analysis
(MCA-102)****Time: Three Hours]****[Maximum Marks: 60**

Note: Question Number 1 is Compulsory. Answer any four from the remaining. Scientific calculators are allowed.

Q.1**(10X2=20 Marks)****(a) When would we not use N-R method?****Solution.**If x_i is the exact root and $0 \times$ is its approximate value of the equation

$$f(x) = 0. \text{ we know that } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

if $f'(x_0)$ is small the error $\frac{f(x_0)}{f'(x_0)}$ will be large and the computation of the root by

this method will be a slow process or may even be impossible. hence the method should not be used in cases where the graph of the function when it crosses the x axis nearly horizontal.

(b) State the principle used in Gauss Jordan method?**Solution.**

Coefficient matrix is transformed into diagonal matrix.

(c) Why Gauss sieidel method is better than Jacobi's iterative method?**Solution.**

since the current values of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration, the convergence in Gauss sieidel method will be more rapid than in Gauss Jacobi method.

(d) When do we apply Lagrange's interpolation?**Solution.**

Lagrange's interpolation formula can be used when the values of "x" are equally spaced or not. It is mainly used when the values are unevenly spaced.

(e) Write the Newton's forward difference interpolation formula.**Solution.**Let the function $y = f(x)$ be a function takes the values $y_0, y_1, y_2, \dots, y_n$ corresponding to the values $x_0, x_1, x_2, \dots, x_n$ where the values of x are equally spaced

Then the Newton's forward difference interpolation formula is given by

$$y_n = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots \dots \dots$$

where $u = \frac{x - x_0}{h}$

(f) What are the advantages of Lagrange's formula over Newton's formula?

Solution.

The forward and backward interpolation formulae of Newton can be used only when the values of the independent variable x are equally spaced and can also be used when the differences of the dependent variable y become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x , the independent variable are equally spaced or not and whether the difference of y become smaller or not.

(g) When do we use Newton's divided differences formula?

Solution.

This is used when the data are unequally spaced.

(h) State Newton's formula to find $f'(x)$, $f''(x)$ & $f'''(x)$ at the point $x = x_n$, using backward difference.

Solution.

At $x = x_0$

For backward difference consider $\Delta = \nabla$ in the following equations

$$f'(x) = \frac{dy}{dx} = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \dots \dots \right]$$

$$f''(x) = \frac{d^2 y}{dx^2} = \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 + \dots \dots \dots \right]$$

$$f'''(x) = \frac{d^3 y}{dx^3} = \frac{1}{h^3} \left[\Delta^3 y_0 - \frac{3}{2} \Delta^4 y_0 + \dots \dots \dots \right]$$

(i) Compare Trapezoidal rule and Simpson's one-third rule for evaluating numerical integration.

Solution.

Trapezoidal rule	Simpson's one-third rule
Any number of Intervals	Number of intervals must be even.
Least accuracy	More accuracy

(j) Taylor series method will be very useful to give some.....for powerful numerical methods such as Runge-Kutta method etc.

Solution.

Initial starting values.

(4X10=40 Marks)

Q.2 Find a real root of the equation $f(x) = x^3 - 3x + 10.6 = 0$, using bisection method correct to three decimal places.

Solution- let $f(x) = x^3 - 3x + 10.6 = 0$

Since $f(1) = 1^3 - 3 \times 1 + 10.6 = 8.6$, which is positive

$$f(2) = 2^3 - 3 \times 2 + 10.6 = 12.6$$

$$f(3) = 3^3 - 3 \times 3 + 10.6 = 28.6$$

$$f(0) = 0^3 - 3 \times 0 + 10.6 = 10.6$$

$$f(-1) = (-1)^3 - 3 \times (-1) + 10.6 = 12.6$$

$$f(-2) = (-2)^3 - 3 \times (-2) + 10.6 = 8.6$$

$$f(-3) = (-3)^3 - 3 \times (-3) + 10.6 = -7.4$$

Therefore, $f(-2)$ is positive and $f(-3)$ is negative, so at least one real root will lie between -2 and -3.

First Iteration- now using Bisection Method, we can take first approximation

$$x_1 = \frac{(-2) + (-3)}{2} = \frac{-5}{2} = -2.5$$

$$f(-2.5) = (-2.5)^3 - 3 \times (-2.5) + 10.6 = 2.475 \text{ which is positive}$$

So root lies between (-2.5) and (-3).

Second Iteration -The second iteration is given by

$$x_2 = \frac{(-2.5) + (-3)}{2} = \frac{-5.5}{2} = -2.75$$

$$f(-2.75) = (-2.75)^3 - 3 \times (-2.75) + 10.6 = -1.946875 \text{ which is negative}$$

So root lies between (-2.75) and (-2.5).

Third Iteration- The Third iteration is given by

$$x_3 = \frac{(-2.75) + (-2.5)}{2} = -2.625$$

$$f(-2.625) = (-2.625)^3 - 3 \times (-2.625) + 10.6 = 0.3871093 \text{ which is positive}$$

So root lies between (-2.625) and (-2.75).

Fourth Iteration- The Fourth iteration is given by

$$x_4 = \frac{(-2.625) + (-2.75)}{2} = -2.6875$$

$$f(-2.6875) = (-2.6875)^3 - 3 \times (-2.6875) + 10.6 = -0.7483886719 \text{ which is negative}$$

So root lies between (-2.6875) and (-2.625) .

Fifth Iteration- The Fifth iteration is given by

$$x_5 = \frac{(-2.6875) + (-2.625)}{2} = -2.65625$$

$$f(-2.65625) = (-2.65625)^3 - 3(-2.65625) + 10.6 = -0.172857666 \text{ which is Negative}$$

So root lies between (-2.65625) and (-2.625) .

Sixth Iteration- The Sixth iteration is given by

$$x_6 = \frac{(-2.65625) + (-2.625)}{2} = -2.640$$

$$f(-2.640) = (-2.640)^3 - 3(-2.640) + 10.6 = 0.120256 \text{ which is Positive}$$

So root lies between (-2.640) and (-2.65625) .

Seventh Iteration- The Seventh iteration is given by

$$x_7 = \frac{(-2.640) + (-2.65625)}{2} = -2.648$$

$$f(-2.648) = (-2.648)^3 - 3(-2.648) + 10.6 = -0.023521792 \text{ which is Negative}$$

So root lies between (-2.648) and (-2.640) .

Eighth Iteration- The Eighth iteration is given by

$$x_8 = \frac{(-2.648) + (-2.640)}{2} = -2.644$$

$$f(-2.644) = (-2.644)^3 - 3(-2.644) + 10.6 = 0.0484940 \text{ which is positive}$$

So root lies between (-2.644) and (-2.648) .

Ninth Iteration- The Seventh iteration is given by

$$x_9 = \frac{(-2.644) + (-2.648)}{2} = -2.646$$

$$f(-2.646) = (-2.646)^3 - 3(-2.646) + 10.6 = 0.012517864 \text{ which is Positive}$$

So root lies between (-2.646) and (-2.648) .

Tenth Iteration- The Seventh iteration is given by

$$x_{10} = \frac{(-2.646) + (-2.648)}{2} = -2.647$$

$$f(-2.647) = (-2.647)^3 - 3(-2.647) + 10.6 = -5.494023 \text{ which is Negative}$$

So root lies between (-2.647) and (-2.646) .

Eleventh Iteration- The Seventh iteration is given by

$$x_{11} = \frac{(-2.647) + (-2.646)}{2} = -2.6465$$

$$f(-2.6465) = (-2.6465)^3 - 3(-2.6465) + 10.6 = 3.513905375 \text{ which is Positive}$$

So root lies between (-2.6465) and (-2.647) .

Twelfth Iteration- The Seventh iteration is given by

$$x_{12} = \frac{(-2.6465) + (-2.647)}{2} = -2.64675$$

$$f(-2.64675) = (-2.64675)^3 - 3(-2.64675) + 10.6 = -9.895625468 \text{ which is Negative}$$

From the above iteration, in eleventh and twelfth approximation the root matches , $f(x) = x^3 - x - 1 = 0$ up to three places of decimal is -2.646 , which is of desired accuracy.

Q.3 Apply Gauss Siedal iteration method to solve the equations:

$$10x + y + 2z = 44, \quad 2x + 10y + z = 51, \quad x + 2y + 10z = 61.$$

Solution – Given system of equation can be written as':

$$x = \frac{1}{10}(44 - y - 2z)$$

$$y = \frac{1}{10}(51 - 2x - z)$$

$$z = \frac{1}{10}(61 - x - 2y)$$

If we start by assuming $y_0 = 0 = z_0$ then we obtain

$$x_1 = \frac{1}{10}(44 - 0 - 0) = 4.4$$

now we substitute $x = 4.4$ and $z_0 = 0$ for y_1 and we obtain

$$y_1 = \frac{1}{10}(51 - 8.8 - 0) = 4.22$$

similarly we obtain

$$z_1 = \frac{1}{10}(61 - 4.4 - 2 \times 4.22) = 4.816$$

now for second approximation, we obtain

$$x = \frac{1}{10}(44 - y - 2z)$$

$$y = \frac{1}{10}(51 - 2x - z)$$

$$z = \frac{1}{10}(61 - x - 2y)$$

$$x_2 = 4.0105$$

$$y_2 = 3.0148$$

$$z_2 = 5.0955$$

Third approximation is given by

$$x = \frac{1}{10}(44 - y - 2z)$$

$$y = \frac{1}{10}(51 - 2x - z)$$

$$z = \frac{1}{10}(61 - x - 2y)$$

$$x_3 = 3.0794$$

$$y_3 = 3.9746$$

$$z_3 = 4.9971$$

Similarly if we proceed up to eight approximation then we obtain

$$x = \frac{1}{10}(44 - y - 2z)$$

$$y = \frac{1}{10}(51 - 2x - z)$$

$$z = \frac{1}{10}(61 - x - 2y)$$

$$x_8 = 3.00$$

$$y_8 = 4.00$$

$$z_8 = 5.00$$

Since $x_7 = x_8$, $y_7 = y_8$ & $z_7 = z_8$ up to two decimal places, the approximate value of the root is $x = 3.00$, $y = 4.00$ and $z = 5.00$.

Q.4 Use the Cholesky's method to solve the equation:

$$2x + y + 4z = 12,$$

$$8x - 3y + 2z = 20,$$

$$4x + 11y - z = 33.$$

Solution-

Let $LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

So that

$$u_{11} = 2, \quad u_{12} = 1, \quad u_{13} = 4$$

$$l_{21}u_{11} = 8 = l_{21} = \frac{8}{2} = 4$$

$$l_{31}u_{11} = 4 = l_{31} = \frac{4}{2} = 2$$

$$l_{21}u_{12} + u_{22} = -3$$

$$u_{22} = -3 - 4 \times 1 = -7$$

$$l_{21}u_{13} + u_{23} = 2$$

$$u_{23} = 2 - 4 \times 4 = -14$$

$$l_{31}u_{12} + l_{32}u_{22} = 11$$

$$l_{32} = \frac{11 - 2 \times 1}{-7} = -\frac{9}{7}$$

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = -1$$

$$u_{33} = -1 - 2 \times 4 - \left(\frac{-9}{7}\right) \times (-14) = -27$$

Since $LUX = B$

Writing $UX = V$, we get

$$LV = B$$

i.e.

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_1 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$v_1 = 12$$

$$4v_1 + v_2 = 20$$

$$v_2 = 20 - 4 \times 12 = -28$$

$$2v_1 - \frac{9}{7}v_2 + v_3 = 33$$

$$v_3 = 33 - 2 \times 12 + \frac{9}{7} \times (-28)$$

$$v_3 = -27$$

Hence the original system $UX = V$ becomes

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$$2x + y + 4z = 12$$

$$\begin{aligned}
-7y - 14z &= -28 \\
-27z &= -27 \\
z &= 1 \\
&= \frac{-28 + 14(1)}{-7} = \frac{-14}{-7} = 2 \\
y &= 2 \\
x &= \frac{12 - 2 - 4 \times 1}{2} = \frac{6}{2} = 3 \\
x &= 3
\end{aligned}$$

Hence the solution is $x = 3$ $y = 2$ $z = 1$ Ans.

Q.5 Compute the value of the definite integral

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) dx$$

by (i) Trapezoidal rule(ii) Simpson's 1/3 Rule (iii) Simpson's 3/8 Rule?

Solution- Solution.

Suppose in this case we divide the range of integration into 12 equal parts by taking $h=1.4-0.2/12=0.1$ now the values of the given function computed for each point of subdivision are as follows:

x_k	$\sin x$	$\log_e x$	e^x	y_k
0.2	0.19867	-1.60944	1.2214	3.02951
0.3	0.29552	-1.20397	1.34986	2.84935
0.4	0.38942	-0.91629	1.49182	2.79753
0.5	0.47943	-0.69315	1.64872	2.8213
0.6	0.56464	-0.51083	1.82212	2.89759
0.7	0.64422	-0.35667	2.01375	3.01464
0.8	0.71736	-0.22314	2.22554	3.16604
0.9	0.78333	-0.10536	2.4596	3.34829
1	0.84147	0	2.71828	3.55975
1.1	0.89121	0.09531	3.004171	3.800071
1.2	0.93204	0.18232	3.32012	4.06984
1.3	0.96356	0.26236	3.6693	4.3705
1.4	0.98545	0.33647	4.0552	4.70418

(i) Trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x) dx = \frac{h}{2} \{y_0 + 2(y_1 + y_2 + y_3 + \dots \dots y_{n-1}) + y_n\}$$

$$l = \frac{0.1}{2} [81.12349] = 4.05617$$

(ii) Simpson's 1/3 Rule

$$\int_a^b f(x) dx \approx \frac{1}{3} h [y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + \dots + 4y_{2n} + y_{2n+1}]$$

$$l = \frac{0.1}{3} [121.53251] = 4.05108$$

(iii) Simpson's 3/8 Rule

$$l = \frac{3 \times 0.1}{8} [108.03098] = 4.05116$$

Q.6 Using Newton Raphson Method determine a real root of the equation

$$f(x) = x^3 - 9x + 1 = 0.$$

Solution-

$$f(x) = x^3 - 9x + 1 = 0 \quad (1)$$

Then differentiating $f(x)$ with respect to x we get

$$f'(x) = 3x^2 - 9 \quad (2)$$

Newton's iterative formula is

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \quad (3)$$

$$\text{Since } f(a) = f(2) = 2^3 - 9 \times 2 + 1 = -9 < 0$$

$$f(b) = f(3) = 3^3 - 9 \times 3 + 1 = 1 < 0 \quad (4)$$

There exists a root of $f(x) = 0$ in the interval $(a, b) = (2, 3)$. clearly the root lies closer to $x = 3$. taking $x_0 = 2.5$ as an initial approximation to the root, we

$$x_{n+1} = x_n - \frac{x_n^3 - 9x_n + 1}{3x_n^2 - 9} \quad (5)$$

$$\therefore x_1 = x_0 - \frac{x_0^3 - 9x_0 + 1}{3x_0^2 - 9} = 2.5 - \frac{2 \cdot 5^3 - 9 \times 2.5 + 1}{3 \cdot 2.5^2 - 9} = 3.1026$$

$$x_2 = x_1 - \frac{x_1^3 - 9x_1 + 1}{3x_1^2 - 9} = 3.1026 - \frac{3.1026^3 - 9 \times 3.1026 + 1}{3 \cdot (3.1026)^2 - 9} = 2.9546;$$

$$x_3 = x_2 - \frac{x_2^3 - 9x_2 + 1}{3x_2^2 - 9} = 2.9546 - \frac{2.9546^3 - 9 \times 2.9546 + 1}{3 \cdot (2.9546)^2 - 9} = 2.9429;$$

$$x_4 = x_3 - \frac{x_3^3 - 9x_3 + 1}{3x_3^2 - 9} = 2.9429 - \frac{2.9429^3 - 9 \times 2.9429 + 1}{3 \cdot (2.9429)^2 - 9} = 2.9428;$$

$$x_5 = x_4 - \frac{x_4^3 - 9x_4 + 1}{3x_4^2 - 9} = 2.9428 - \frac{2.9428^3 - 9 \times 2.9428 + 1}{3 \cdot (2.9428)^2 - 9} = 2.94282;$$

Since $x_4 = x_5$ up to five decimal places, the approximate value of the root is 2.9428.

Or

$$f(x) = x^3 - 9x + 1 = 0 \quad (1)$$

Then differentiating $f(x)$ with respect to x we get

$$f'(x) = 3x^2 - 9 \quad (2)$$

Newton's iterative formula is

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \quad (3)$$

$$\text{Since } f(a) = f(2) = 2^3 - 9 \times 2 + 1 = -9 < 0$$

$$f(b) = f(3) = 3^3 - 9 \times 3 + 1 = 1 < 0 \quad (4)$$

There exists a root of $f(x) = 0$ in the interval $(a, b) = (2, 3)$. clearly the root lies closer to $x = 3$. taking $x_0 = 3$ as an initial approximation to the root, we

$$x_{n+1} = x_n - \frac{x_n^3 - 9x_n + 1}{3x_n^2 - 9} = \frac{2x_n^3 - 1}{3x_n^2 - 9} \quad (5)$$

$$\therefore x_1 = \frac{2x_0^3 - 1}{3x_0^2 - 9} = \frac{2 \cdot 3^3 - 1}{3 \cdot 3^2 - 9} = \frac{53}{18} = 2.94$$

$$x_2 = \frac{2x_1^3 - 1}{3x_1^2 - 9} = \frac{50.05521262}{17.00925925} = 2.94282142;$$

$$x_3 = \frac{2x_2^3 - 1}{3x_2^2 - 9} = \frac{49.97083229}{16.98059389} = 2.942820058$$

Since $x_2 = x_3$ up to five decimal places, the approximate value of the root is 2.94282.

Note- for the above solution, check the following algorithm for minimum approximations.

Algorithm

The steps of the Newton-Raphson method to find the root of an equation $f(x) = 0$ are-

1. Evaluate $f'(x)$ symbolically.
2. Use an initial guess of the root, x_i , to estimate the new value of the root, x_{i+1} , as

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

3. Find the absolute relative approximate error $|\epsilon_a|$ as $|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$
4. Compare the absolute relative approximate error with the pre-specified relative error tolerance, ϵ_s . If $|\epsilon_a| > \epsilon_s$, then go to Step 2, else stop the algorithm. Also, check if the number of iterations has exceeded the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user.

Q.7 Calculate by Simpson's 1/3 method and Simpson's 3/8 method an approximate value of $\int_{-3}^3 x^4 dx$ by taking seven equidistant ordinates. Compare it with the exact value obtained by using the Trapezoidal rule.

Solution-

Here the range of integration $(-3, 3)$ is subdivided into six equal parts by seven equidistant ordinates. So $h = \frac{3 - (-3)}{6} = 1$. now the values of the given function $y = x^4$ computed for each point of subdivision are as given below:

x_k	$y_k = x^4$	Simpson's 1/3 rule	Simpson's 3/8 Rule
$x_0 = -3$	$y_0 = 81$	$y_0 = 81$	$y_0 = 81$
$x_1 = x_0 + h = -2$	$y_1 = 16$	$4y_1 = 64$	$2y_1 = 32$
$x_2 = x_0 + 2h = -1$	$y_2 = 1$	$2y_2 = 2$	$2y_2 = 2$
$x_3 = x_0 + 3h = 0$	$y_3 = 0$	$4y_3 = 0$	$2y_3 = 0$
$x_4 = x_0 + 4h = 1$	$y_4 = 1$	$2y_4 = 2$	$2y_4 = 2$
$x_5 = x_0 + 5h = 2$	$y_5 = 16$	$4y_5 = 64$	$2y_5 = 32$
$x_6 = x_0 + 6h = 3$	$y_6 = 81$	$y_6 = 81$	$y_6 = 81$
	Total	294	230

Therefore by Simpson's 1/3 rule

$$\int_{-3}^3 x^4 dx = \frac{1}{3}[294] = 98 \quad [\because h = 1]$$

the exact value of the integral

$$\int_{-3}^3 x^4 dx = \left[\frac{x^5}{5} \right]_{-3}^3 = \frac{1}{5}[(3)^5 - (-3)^5] = 97.2$$

again by trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x) dx = \frac{h}{2} \{y_0 + 2(y_1 + y_2 + y_3 + \dots \dots y_{n-1}) + y_n\}$$

$$\int_{-3}^3 x^4 dx = \frac{1}{2}[230] = 115$$

above the result show that the trapezoidal rule does not give an accurate result in the present case while Simpson's rule gives the value which is much nearer to the exact value .

Q.8 Solve $\frac{dy}{dx} = \frac{1}{x+y}$ for $x = 0.5$ by using Runge's Kutta Method with $x_0 = 0, y_0 = 1$ (take $h = 0.5$).

Solution –

$$\text{here } x_0 = 0, y_0 = 1, h = 0.5 \text{ and } f(x, y) = \frac{1}{x+y}$$

For the first interval we have

$$S_1 = hf(x_0, y_0) = 0.5 \left(\frac{1}{0+1} \right) = 0.5$$

$$S_2 = hf \left(x_0 + \frac{h}{2}, y_0 + \frac{S_1}{2} \right) = 0.5 f(0.25, 1.25)$$

$$= 0.5 \left(\frac{1}{0.25 + 1.25} \right) = 0.5 \frac{1}{1.5} = 0.33333$$

$$S_3 = hf \left(x_0 + \frac{h}{2}, y_0 + \frac{S_2}{2} \right) = 0.5 f \left(0.25, \frac{7}{6} \right)$$

$$= 0.5 \left(\frac{1}{0.25 + \frac{7}{6}} \right) = 0.35294$$

$$S_4 = hf(x_0 + h, y_0 + S_3) = 0.5 f(0.5, 1.35294) = 0.5 \left(\frac{1}{0.5 + 1.35294} \right) = 0.26984$$

so the value of y at $x_1 = 0.5$ is given by

$$\begin{aligned} y_1 &= y_0 + \frac{1}{6}(S_1 + 2S_2 + 2S_3 + S_4) \\ &= 1 + \frac{1}{6}(0.5 + 0.6667 + 0.7058 + 0.2698) \\ &= 1 + \frac{1}{6}(2.1423) \end{aligned}$$

$$= 1 + 0.35705 = 1.35705$$

$$\therefore \text{at } x = 0.5, y = 1.35705.$$

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